

and products as it is amongst the lowest, the exact value of the coefficient cannot be held to have been satisfactorily determined. But M. Gogau has entirely neglected these terms, whereas in my investigation I retained them; hence his result is not sufficiently complete.

It is further to be noted that this elaborate investigation of M. Gogau, which reflects so much credit on its author, forms only the smaller portion of the complete work of determining the true value of this term of long period due to *Mars*, for, like Delaunay, he has entirely omitted to consider the difficult but most highly important effects arising from the direct action of the Sun on the perturbations in the motion of the Moon due to the direct action of *Mars*. But, as Hansen found, their indirect effect may be much more important than the direct effect of the planet. So until this additional portion of the complete value of the coefficient of this term has been computed, it cannot be said that the value of this term has been shown to be insensible, as M. Gogau believed he had done in his fine Memoir.

London :
1882, February 15.

On the Correction to the Horizontal Diameter of the Moon, from the Observations made between 1851 and 1858. By James Campbell, Esq., and E. Neison, Esq.

In his Paper in the *Monthly Notices* for December 1881, p. 64, Mr. Stone states that he deduces from the Greenwich Observations for the years 1851-58 a very different value for the correction to the tabular semi-diameter to that given by us in our Paper as the correction deduced by one of us (Neison). It is necessary for us, therefore, to publish the details of this investigation made by Mr. Neison some years ago.

The determination of the correction to the semi-diameter may be divided into two portions—one, 1851-55, when it is necessary to correct the value of the semi-diameter given in the *Nautical Almanac* for the errors in Burckhardt's tables, by applying to the parallax and semi-diameter the corrections deduced by Adams, and given in the *Nautical Almanac* for 1856; and the other for the years 1856-58, where no such correction is required, as Adams's corrected parallax and semi-diameter were used in the *Nautical Almanac*.

Let

S_a = Adams's original value of the mean semi-diameter of the Moon,

δS_a = Correction to this value given by the Greenwich Observations,

S'_h = Tabular horizontal semi-diameter of the Moon (apparent),

S'_v = Tabular vertical semi-diameter of the Moon (apparent);

and put

$\Delta S'_a$ = Adams's correction to Burckhardt's tabular parallax,

$\Delta P'_a$ = Adams's correction to Burckhardt's tabular semi-diameter,

C_b = Observed correction to Burckhardt's horizontal semi-diameter,

C_a = Observed correction to Adams's horizontal semi-diameter.

Then

$$C_a = C_b + \Delta S'_a \times \frac{S'_h}{S'_v},$$

and

$$\begin{aligned} \delta S_a &= C_a \times \frac{S}{S'_h} = \left(C_b + \Delta S'_a \times \frac{S'_h}{S'_v} \right) \frac{S}{S'_h} \\ &= C \times \frac{S}{S'_h} + 0.2725 \Delta P'_a \times \frac{S}{S'_v}; \end{aligned}$$

where

S = Mean tabular semi-diameter = $932\frac{5}{5}$ approximately.

In both Burckhardt's tables and in Adams's original value, the semi-diameter is obtained by multiplying the parallax by the constant 0.2725 . Burckhardt's value for the constant of parallax is $(57' 0'' 5)$, so that his value for the mean semi-diameter is

$$(57' 0'' 5) \times 0.2725 = 15' 32'' 09.$$

The constant of parallax adopted by Adams is different, however, and is explicitly stated by him in the introduction to his supplementary tables (*Naut. Alm.*, 1856) to be $(57' 2'' 325)$, hence the value of Adams's original mean semi-diameter is

$$(57' 2'' 325) \times 0.2725 = 15' 32'' 57,$$

or greater than Burckhardt's by $0'' 48$.

From the different volumes of the Greenwich Observations and the corresponding volumes of the *Nautical Almanac* are derived the quantities:—

| No. | Date. | Approximate | | $\Delta P'_a$ | Mean | | Corr. applied in N.A. to Burck- hardt's Tab. |
|-----|---------------|-----------------|------------------|---------------|--------------------------------|---------------------------------|--|
| | | Hor. Semi-d. | Vert. Semi-d. | | semi-dia. Hor. semi-dia. | semi-dia. Vert. semi-dia. | |
| 1 | 1851, Feb. 15 | 72.4 | 1.019 | 12.88 | 0.915 | +0.8 | +.00 |
| 2 | May 14 | 68.1 | .964 | 13.70 | 0.968 | +0.7 | +.00 |
| 3 | June 13 | 68.5 | .930 | 13.61 | 1.003 | +3.0 | +.00 |
| 4 | July 12 | 67.4 | .910 | 13.84 | 1.025 | +4.1 | +.00 |
| 5 | Aug. 11 | 63.9 | .893 | 14.58 | 1.044 | +3.6 | +.00 |
| 6 | Sept. 9 | 62.1 | .889 | 15.03 | 1.049 | +1.8 | +.00 |
| 7 | 1852, March 5 | 70.6 | 1.013 | 13.22 | .921 | +4.0 | +.00 |

| No. | Date. | Approximate | | Mean | Mean | $\Delta P'^a$ | Corr. applied in N.A. to Burch- hardt's Tab. | Value. |
|-----|---------------|-----------------|------------------|--------------------------------|---------------------------------|---------------|--|--------|
| | | Hor. Semi-d. | Vert. Semi-d. | semi-dia. Hor. semi-dia. | semi-dia. Vert. semi-dia. | | | |
| 8 | 1852, May 3 | 70.8 | 1.004 | 13.17 | .929 | + 2.5 | .00 | +.00 |
| 9 | Nov. 26 | 65.2 | .902 | 14.31 | .934 | + 1.5 | .00 | +.00 |
| 10 | 1853, Mar. 24 | 68.3 | .995 | 13.65 | .950 | + 2.6 | .19 | +.19 |
| 11 | May 22 | 74.2 | 1.013 | 12.57 | .921 | - 2.1 | .21 | +.21 |
| 12 | Sept. 16 | 65.1 | .940 | 14.33 | .993 | + 1.8 | .18 | +.18 |
| 13 | Oct. 16 | 62.6 | .914 | 14.91 | .9020 | + 2.4 | .17 | +.17 |
| 14 | 1854, Feb. 12 | 66.2 | .921 | 14.09 | 1.013 | + 0.4 | .18 | +.18 |
| 15 | June 10 | 75.6 | 1.006 | 12.34 | .927 | - 1.6 | .22 | +.22 |
| 16 | Sept. 6 | 68.8 | .996 | 13.56 | .937 | + 2.8 | .19 | +.19 |
| 17 | 1855, March 3 | 62.5 | .905 | 14.93 | 1.030 | - 1.0 | .17 | +.17 |
| 18 | July 28 | 76.9 | 1.004 | 12.13 | .929 | + .8 | .22 | +.22 |
| 19 | Sept. 25 | 69.1 | 1.009 | 13.50 | .924 | + 1.5 | .19 | +.19 |

The observations made by computers or other less experienced observers are marked with an asterisk, and, as stated on p. 443 of our paper, are rejected as liable to unknown error.

| No. | Obs. Corr. to Naut. Alm. | C _b | C _b × $\frac{S}{S'h}$ | 0.2725 | $\Delta P'^a$ | δS_a |
|-----|-----------------------------|----------------|----------------------------------|--------------------------|--------------------------|--------------|
| | | | | $\times \frac{S'}{S'_v}$ | $\times \frac{S'}{S'_v}$ | |
| 1 | -.17 | -.17 | -.2'20 | " | + 0.20 | -.2'00 |
| 2 | -.18 | -.18 | -.2'47 | " | + 0.19 | -.2'28 |
| 3 | -.18 | -.18 | -.2'45 | " | + 0.82 | - 1.63* |
| 4 | -.20 | -.20 | -.2'77 | " | + 1.19 | - 1.58 |
| 5 | -.17 | -.17 | -.2'58 | " | + 1.03 | - 1.55 |
| 6 | -.19 | -.19 | -.2'86 | " | + 0.52 | - 2.34 |
| 7 | -.23 | -.23 | -.3'04 | " | + 1.00 | - 2'04 |
| 8 | -.29 | -.29 | -.3'82 | " | + .63 | - 3.19 |
| 9 | -.29 | -.29 | -.4'15 | " | + .43 | - 3.72 |
| 10 | -.05 | -.24 | -.3'27 | " | + .67 | - 2.60 |
| 11 | -.19 | -.40 | -.5'03 | " | - .52 | - 5.55 |
| 12 | -.07 | -.25 | -.3'58 | " | + .48 | - 3.10 |
| 13 | -.15 | -.32 | -.4'77 | " | + .67 | - 4.10 |
| 14 | +.10 | -.17 | -.2'39 | " | + 1.1 | - 2.28 |
| 15 | +.06 | -.06 | -.74 | " | - .41 | - 1.15* |
| 16 | +.10 | -.18 | -.2'44 | " | + .71 | - 1.73 |
| 17 | +.09 | -.08 | -.1'19 | " | - .28 | - 1.47* |
| 18 | +.01 | -.12 | -.1'45 | " | + .20 | - 1.25 |
| 19 | -.05 | -.24 | -.3'24 | " | + .38 | - 2.86 |

Y

Hence we have

| | | | | |
|------|----------|---------|-------------------------------|--------|
| 1851 | 6 obs. = | - 1°90. | Excluding computers, 5 obs. = | - 1°95 |
| 1852 | 3 | - 2°97. | 3 | - 2°97 |
| 1853 | 4 | - 3°84. | 4 | - 3°84 |
| 1854 | 3 | - 1°72. | 2 | - 2°01 |
| 1855 | 3 | - 1°86. | 2 | - 2°06 |

For the year 1856 the value employed in the *Nautical Almanac* was derived from Adams's parallax multiplied by the constant 0°2725. The tables require no correction, and the corrections to the *Nautical Almanac* given by the Greenwich Observations yield directly the correction of the semi-diameter.

For this year, then, we have

| Date. | $S'h$ | $\frac{S}{S'h}$ | C_d | $C_d \times \frac{S}{S'h}$ |
|--------------|-------------------|-----------------|-------|----------------------------|
| 1856, May 19 | 68°4 ^s | 13°63 | - .07 | - .95 |
| Aug. 15 | 72°2 | 12°93 | - .10 | - 13* |
| Oct. 13 | 70°5 | 13°24 | - .00 | - .00 |

Hence

$$1856 \quad 3 \text{ obs.} = - 0°36. \quad \text{Excluding computers, 2 obs.} = - 48.$$

For the two years 1857-58 the value employed in the *Nautical Almanac* was derived from Adams's parallax multiplied by the constant 0°273114. Hence the mean diameter, instead of being 15°32°57, was

$$(57 2°325) \times 0°273114 = 15 34°68.$$

or 2°11 larger. For these years we have

| Date. | $S'h$ | $\frac{S}{S'h}$ | C_d | $C_d \times \frac{S}{S'h}$ |
|---------------|-------------------|-----------------|-------|----------------------------|
| 1857, May 8 | 65°0 ^s | 14°36 | - .11 | - 1°58 |
| Nov. 1 | 73°3 | 12°73 | + .20 | + 2°55* |
| Dec. 30 | 78°7 | 11°84 | + .11 | + 1°30 |
| 1858, Feb. 27 | 66°0 | 14°13 | - .02 | - 0°28 |
| July 25 | 66°9 | 13°95 | + .05 | + 0°69 |
| Sept. 22 | 64°6 | 14°45 | + .10 | + 1°45 |
| Oct. 22 | 70°2 | 13°30 | + .15 | + 2°00 |
| Nov. 20 | 75°6 | 12°34 | + .10 | + 1°23 |

Hence

$$1857 \quad 3 \text{ obs.} = + 0°76. \quad \text{Excluding computers, 2 obs.} = - 0°14$$

$$1858 \quad 5 \quad + 1°02. \quad 5 \quad + 1°02$$

To reduce all the results to the same value of the semi-diameter—namely, that commonly called Adams's value, and used in the *Nautical Almanac* for 1857-61—those for the years 1851-56 must be increased by $+2''\cdot 11$. Hence the entire series become

| | | | | | |
|------|--------|--------------------|---------------------|--------|------------------|
| 1851 | 6 obs. | $= +0''\cdot 21$. | Excluding computers | 5 obs. | $= +0''\cdot 16$ |
| 1852 | 3 | $-0\cdot 86$. | | 3 | $-0\cdot 86$ |
| 1853 | 4 | $-1\cdot 73$. | | 4 | $-1\cdot 73$ |
| 1854 | 3 | $+0\cdot 39$. | | 2 | $+0\cdot 10$ |
| 1855 | 3 | $+0\cdot 25$. | | 2 | $+0\cdot 05$ |
| 1856 | 3 | $+1\cdot 75$. | | 2 | $+1\cdot 63$ |
| 1857 | 3 | $+0\cdot 76$. | | 2 | $-0\cdot 14$ |
| 1858 | 5 | $+1\cdot 02$. | | 5 | $+1\cdot 02$ |

Multiplying each of these by the number of observations and taking the means, we have

$$30 \text{ obs.} = +0''\cdot 21. \quad 24 \text{ obs.} = -0''\cdot 01$$

As explicitly stated by us, the observations of the computers were excluded; hence we have for the correction to the tabular semi-diameter $= -0''\cdot 01$, or for the observed semi-diameter

$$(15^{\circ} 34' 68) - (-0\cdot 01) = 15^{\circ} 34' 69.$$

This result is exactly the same as that obtained by a second computation made by using the corrections to the actual diameter, instead of the semi-diameter, by which the necessity of dividing the corrections by two was avoided.

The only other point raised by Mr. Stone which need be considered is, how far we are right in believing the large apparent value of the parallactic inequality obtained from the observations of the years 1846-50 to be probably due to the repeated changes in the observers during those years. Previous to 1846 Mr. Ellis seems to have made the greater number of observations, whilst subsequent to that year he made very few; and we point out that as Mr. Ellis seems to have made the correction to the diameter of the Moon nearly $3''$ greater than the other observers, the cessation of his observations must have tended to have raised the apparent value of the parallactic inequality.

This is questioned by Mr. Stone, who from *four* observations of the horizontal diameter of the Moon made by Mr. Ellis, as compared with twenty-three by other observers, shows that Mr. Ellis's value only exceeds those of the other observers by $1''\cdot 94$.* But during the period covered by the observations quoted by Mr. Stone the instrument with which they were made was changed, and to obtain comparable results it is necessary to ex-

* These are taken direct from Mr. Stone's figures. There are two or three small errors in this portion of his paper.

clude the year 1851 and restrict ourselves to the consideration of the years prior to 1851, when the observations were throughout made with the old Transit Instrument. Employing, then, the twenty observations made between 1843 and 1850, we find from *three* observations of Mr. Ellis, as compared with seventeen by other observers, that the mean of the former exceeds the mean of the latter by $2''\cdot62$. This result practically accords with those obtained by us; but we demur to the idea that any trustworthy results can be obtained from the comparison of only three or four observations of Mr. Ellis with other data. Our own statement that the correction to the lunar diameter according to Mr. Ellis exceeded that according to the three other observers, Messrs. Henry, Rogerson, and Dunkin, by nearly $3''$, was founded on the direct comparison of all their observations, made during the years 1843-50, taking Mr. Henry, who regularly observed throughout the entire period, as the standard observer. This comparison yielded the following results:—

Correction to diameter.

| | | |
|------------------|------------------|--|
| Henry - Rogerson | = $+0\cdot1''$. | From 174 obs. of the limb, and 8 obs. of diameter. |
| Henry - Dunkin | = $+0\cdot2''$. | From 147 obs. of the limb. |
| Henry - Ellis | = $+2\cdot9''$. | From 325 obs. of the limb, and 8 obs. of diameters. |

As in the determination of the parallactic inequality, the correction to the assumed diameter enters multiplied by a factor whose mean value may be taken as $0\cdot75$, it follows that the value found from Mr. Ellis's observations requires the correction $2''\cdot9 \times 0\cdot75 = 2''\cdot17$ to bring it into accord with the value according to Mr. Henry's observations. Hence, owing to the cessation of Mr. Ellis's observation in 1846, it follows that the value of the parallactic inequality found for that year requires a correction of over $1''$ to bring it into harmony with the values found for previous years. Similar corrections must also be applied to the results for the other subsequent years.

This correction does not completely remove the discordance between the results obtained from the observations made during the short period 1846-50, and the long series prior to 1846. But it weakens its weight enormously. Further examination would probably reveal other causes; but the existence of this discordance is not of sufficient importance to render its removal worthy of the labour involved. To us it seems that this admitted discordance of four or five years in a period of one hundred and twenty cannot be held to be serious, when it is known to be coincident in period with the introduction of more inexperienced observers, and vanishes as soon as these observers become more experienced, as in the period 1851-58.